be cambered and twisted in such a way that the desired lift distribution is obtained for a given speed, altitude, and load factor, the ASW requires a considerably larger amount of twist than the FSW. Consequently, the ASW will experience larger drag penalties in off-design conditions. Aeroelastic tailoring can, of course, alleviate this problem in that the elastic wash-out occurring on the ASW when lift is increased can partly compensate the corresponding aerodynamic wash-in. However, aerodynamic/elastic matching can be obtained at only one particular equivalent airspeed.

Conclusion

When vortex drag is minimized for a given wing weight a nonelliptical spanwise lift distribution form is obtained that shows reduced loading on the outer wing and correspondingly increased wing loading at the root. This distribution form produces 5% less vortex drag than the elliptical distribution. The forward-swept trapezoidal plane wing with a taper ratio of 0.2 and −30 deg leading-edge sweep has a lift distribution very close to the optimum distribution form, while the lift distribution of the corresponding aft-swept wing differs considerably from this. The ASW will, therefore, experience a higher deterioration of the spanwise lift distribution at off-design conditions. In contrast, an untwisted FSW fitted with an aeroisoclinic behavior[3] will exhibit an almost ideal lift distribution at all subsonic points of the flight envelope.

References

[1]Munk, M.M., "The Minimum Induced Drag of Aerofoils," NACA Rept. 121, 1921.

[2]Weißinger, J., "Über eine Erweiterung der Prandtlschen Theorie der tragenden Linie," *Mathematische, Nachrichten,* Vol. 2, Jan./Feb. 1949, pp. 45-106.

[3]Weisshaar, T.A., "Aeroelastic Tailoring of Forward Swept Wings," *Proceedings of 21st Structures, Structural Dynamics, and Materials Conference,* Seattle, Wash., May 12-14, 1980, pp. 761-770.

[4]Klein, A. and Viswanathan, S.P., "Approximate Solution for Minimum Induced Drag of Wings with Given Structural Weight," *Journal of Aircraft,* Vol. 12, Feb. 1975, pp. 124-126.

AIAA 81-4188

Dynamic Stress in a Towing Wire due to Forced Acceleration

C. Matuk*
University of Luleå, Luleå, Sweden

Introduction

WIRES used to tow bodies in air or water occasionally break. The reason is often a combination of static and dynamic stresses which causes the total stress to reach the tensile strength somewhere in a wire. The static stress is due to gravity and aerodynamic or hydrodynamic drag, and reaches its maximum at the towing end. The dynamic stress can be caused by accelerations of the towing vehicle. In this Note dynamic stress due to such accelerations is studied in the case of rectilinear motion (see Fig. 1).

The diameter of the wire is small compared to the wavelengths of the dominant Fourier components of the elastic waves. Also, internal friction in the wire is neglected. Thus, the one-dimensional wave equation applies. The towed body is represented by a rigid mass.

Received June 24, 1980; revision received March 9, 1981.

*Department of Mechanical Engineering.

Glauert[1] was one of the first who studied the shape of a towing wire. The case of a very light wire was investigated by Landweber and Protter.[2] The inclusion of wave propagation in towing came later, e.g., see Narkis.[3] Genin, Citron, and Huffman[4] considered dynamic stress caused by a variable force applied to the towed body.

Theory

Consider a towing vehicle which first travels at a constant velocity and then accelerates to a higher constant velocity. The towing wire is characterized by its density ρ, Young's modulus E, cross-sectional area A, and length L. The towed body is characterized by its mass m.

A reference frame moving with the original constant velocity of the towing vehicle is used. The longitudinal displacement in the wire is denoted by $u(x,t)$, where x is the position and t the time. Thus, the velocities appearing in this Note are velocities in excess of the original vehicle velocity. Also, the stress is dynamic stress in excess of the original static stress. The dynamic stress $\sigma = E\partial u/\partial x$ is related to the particle velocity $v = \partial u/\partial t$ by

$$\partial\sigma/\partial t = E\partial v/\partial x \tag{1}$$

The particle velocity satisfies the wave equation

$$\partial^2 v/\partial t^2 = c^2 \partial^2 v/\partial x^2 \tag{2}$$

where $c = (E/\rho)^{1/2}$ is the elastic wave speed.

Initially, the wire is free from dynamic stress and at rest in the moving reference frame. Thus, the initial conditions are

$$\sigma(x,0) = 0, \quad v(x,0) = 0, \quad \partial v(x,0)/\partial t = 0, \quad 0<x<L \tag{3}$$

The dynamic stress at the towed end is related to the velocity of the towed body through Newton's second law. Also, the velocity at the towing end is the same as the velocity v_T of the towing vehicle. Thus, the boundary conditions are

$$m\partial v(0,t)/\partial t = A\sigma(0,t), \qquad v(L,t) = v_T(t), \qquad t>0 \tag{4}$$

The following dimensionless parameters are convenient to introduce here:

$$\xi = x/L, \quad \tau = t/T_0, \quad V = v/v_0, \quad V_T = v_T/v_0$$
$$\mu = m/M, \quad \lambda = T_0/t_0, \quad S = \sigma/\sigma_0 \tag{5}$$

where $T_0 = L/c$ is the transit time for a wave through the wire and $M = \rho AL$ is the mass of the wire. t_0 is the time during which the towing vehicle accelerates and v_0 is the final velocity increase. Also, $\sigma_0 = m(v_0/t_0)/A$ is defined as reference stress. μ can be interpreted as a dimensionless mass of the towed body and λ as a dimensionless length of the wire.

Equations (5) are used to make Eqs. (1-4) dimensionless. Laplace transformation $[\mathcal{L}\{f(\xi,\tau)\} = \bar{f}(\xi,s)]$ then gives

$$\mu\lambda s\bar{S} = \partial\bar{V}/\partial\xi \tag{6}$$

$$\partial^2\bar{V}/\partial\xi^2 - s^2\bar{V} = 0 \tag{7}$$

$$s\bar{V}(0,s) = \lambda\bar{S}(0,s), \qquad \bar{V}(1,s) = \bar{V}_T(s) \tag{8}$$

The solution of Eqs. (6) and (7), with the boundary conditions in Eqs. (8), gives the Laplace transform $\bar{S}$ of the dynamic

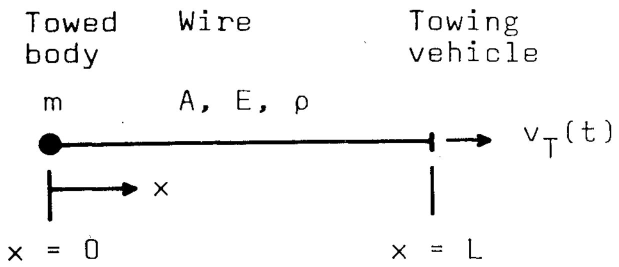

Fig. 1 Model of towing system.

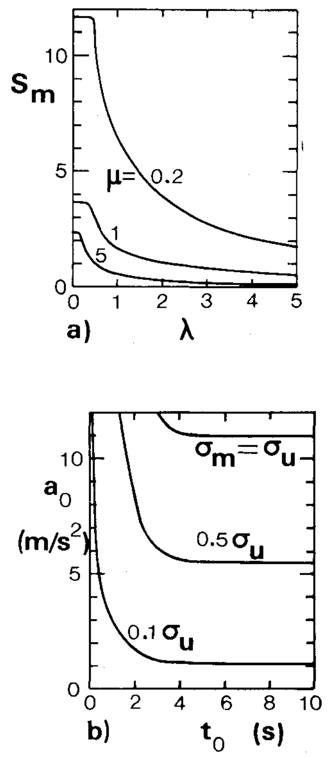

Fig. 2 S_m **as a function of** μ **and** λ **(a) and** a_0 **as a function of** t_0 **for different values of** σ_m **in a particular towing case (b)**

stress in the wire as

$$\mu\lambda\tilde{S}=\tilde{V}_T\sum_{n=0}^{\infty}(-1)^n\exp[-(2n+1-\xi)s]$$
$$\times[(1-\mu s)/(1+\mu s)]^n+\tilde{V}_T\sum_{n=0}^{\infty}(-1)^{n+1}$$
$$\times\exp[-(2n+1+\xi)s][(1-\mu s)/(1+\mu s)]^{n+1} \quad (9)$$

Application to a Rectangular Acceleration Pulse

Let the acceleration of the towing vehicle be a rectangular pulse with amplitude $a_0=v_0/t_0$ and duration t_0. The corresponding velocity in dimensionless form can be expressed as

$$V_T(\tau)=\begin{cases}\tau, & 0\leq\tau<1/\lambda\\ 1, & 1/\lambda\leq\tau\end{cases} \quad (10)$$

The reference stress σ_0 represents the stress caused by the acceleration pulse in a massless wire. The dimensionless dynamic stress $S=\sigma/\sigma_0$ due to this acceleration pulse is obtained by substitution of the velocity [Eq. (10)] into the Laplace inversion of Eq. (9). The mathematical details are presented in Ref. 5.

The dynamic state of stress $S(\xi,\tau)$ depends only on the towed mass μ and the wire length λ. Three-dimensional surfaces $S=S(\xi,\tau)$, also presented in Ref. 5, were computed for values of μ and λ around unity, which are common in practical situations. In all of the cases studied, the maximum dynamic stress S_m occurred at the towing end $\xi=1$, a result which is not evident beforehand. A summary of these results is presented in Fig. 2a in which S_m is shown vs μ and λ. S_m is large for short wire lengths λ because many reflexions of the wave front occur at the towed body and the towing vehicle before the acceleration is ended.

A linear elastic spring can be used as an approximate model for a short wire. The maximum dynamic stress according to this model is $S_m=2(1+1/\mu)$. This result agrees with the result in Fig. 2a for small values of λ, which provides a check of the numerical results.

The results of Fig. 2a can be applied to different practical cases. Consider a towing case in which the wire is of high-strength steel ($\sigma_u=2000$ MPa, $\rho=7.8\times10^3$ kg/m^3, and $E=200$ GPa) and has dimensions $L=6.4$ km and $A=1$ mm^2, and the towed body has mass $m=50$ kg. For this case we use the curve with $\mu=1$ in Fig. 2a to obtain Fig. 2b. In the latter, combinations of towing vehicle acceleration a_0 and acceleration time t_0 which give rise to different maximum dynamic stresses σ_m are shown.

Conclusions

The model in this Note represents a rectilinear wire without damping. In reality, when the towing vehicle travels in a horizontal straight path, the wire adopts a curved shape. Also, damping occurs in the material of the wire and in the interaction between the wire and the surrounding medium. Curved shape and damping tend to reduce the value of the maximum dynamic stress S_m compared with the value obtained here. Therefore Fig. 2 can be considered as a conservative estimate which should have practical use.

Maximum static stress and maximum dynamic stress occur at the towing end. Thus, the maximum total stress occurs at the towing end.

Failure occurs if σ_m in Fig. 2b exceeds the difference between the tensile strength σ_u of the wire and the static stress at the towing end. Notice in particular the horizontal asymptotes. They imply that there is no risk of failure if the acceleration is below a critical value regardless of the duration of the acceleration.

Acknowledgment

This investigation was supported by the Swedish Defense Material Administration (FMV).

References

[1] Glauert, H., "The Form of a Heavy Flexible Cable used for Towing a Heavy Body below an Aeroplane," Aeronautical Research Committee, London, R&M No. 1592, Feb. 1934.

[2] Landweber, L. and Protter, M.H., "The Shape and Tension of a Light Flexible Cable in a Uniform Current," *Journal of Applied Mechanics,* Vol. 14, June 1947, pp. 121-126.

[3] Narkis, Y., "Deployment Forces in Towing Systems," *Journal of Aircraft,* Vol. 15, Feb. 1978, pp. 123-124.

[4] Genin, J., Citron, S.J., and Huffman, R.R., "Coupling of Longitudinal and Transverse Motion of a Flexible Cable in a Uniform Flow Field," *Journal of the Acoustical Society of America,* Vol. 52, July 1972, pp. 438-440.

[5] Matuk, C., "Stress in a Towing Wire due to Accelerations of the Towing Vehicle," Dept. of Mechanical Engineering, University of Luleå, Sweden, Technical Rept. 1979:33T, May 1979.